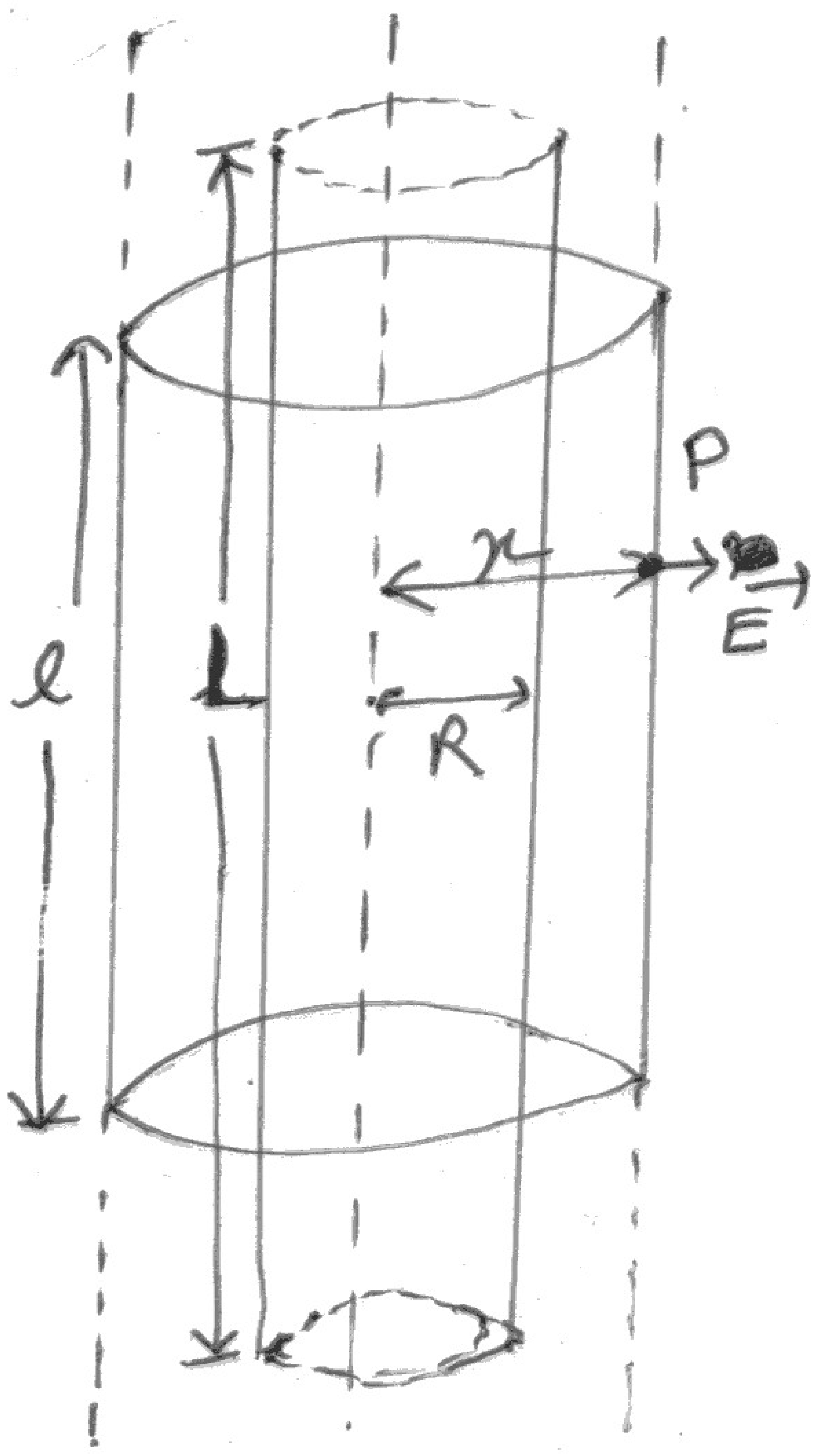
Electric field due to a charged cylindrical conductor

Let there be a cylindrical conductor of infinite length and radius R which is uniformly charged with charge per unit length λ. Since the cylinder is conducting, so the total charge resides on the outer surface of the cylinder.

To find the electric field at a point P at distance x from the axis when the point P lies (1) outside (2) on the surface (3) inside the cylinder.

Consider a coaxial cylinder of length 'l' and radius x as the Gaussian surface. Since the cylinder is uniformly charged, therefore the electric field intensity at any point on the curved surface of Gaussian surface will be the same and directed normal to the surface in outward direction.



Thus the electric flux contribution through the plane surfaces of the Gaussian surface will be zero. Only the electric flux through the curved surface of the Gaussian surface will contribute. The area of curved surface is $2\pi x l$.

Hence the electric flux through the curved surface of Gaussian surface is

$$\phi = \iint \vec{E} \cdot \vec{dS} = \iint E\, dS$$

$$= E \times 2\pi x l$$

Case(i) :- When the point P lies outside the cylinder. (i.e. $x > R$) - According to Gauss's law, the electric flux through the Gaussian surface

$$\phi = \frac{1}{\epsilon_0} \lambda l$$

Since the charge inside the Gaussian surface = charge on length l of the cylinder

$= \lambda l$

$$\therefore \quad E \times 2\pi x l = \frac{1}{\epsilon_0} \lambda l$$

or
$$\boxed{E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x}} \qquad \text{——— (1)}$$

Now to calculate the electric potential at point P, we find the potential difference between the point P and any reference point outside the cylinder.

If the distance of reference point from the axis of cylinder is x_1 and the electric potential is V_{x_1} (known), then the potential difference between the reference point and the point P is

$$V_x - V_{x_1} = -\int_{x_1}^{x} E\,dx = -\int_{x_1}^{x} \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x}\,dx$$

$$= -\frac{1}{2\pi\epsilon_0} \lambda (\log_e x - \log_e x_1)$$

Hence $V_x = V_{x_1} + \frac{\lambda}{2\pi\epsilon_0} \log_e x_1 - \frac{\lambda}{2\pi\epsilon_0} \log_e x$

or
$$\boxed{V_x = -\frac{\lambda}{2\pi\epsilon_0} \log_e x + \text{Constant}} \qquad \text{——— (2)}$$

where $V_{x_1} + \frac{\lambda}{2\pi\epsilon_0} \log_e x_1 = \text{Constant}$. ——— (3)

<u>Case (ii)</u> :– When the point P lies on the surface of cylinder i.e. $x = R$,

From $\hat{Eq}$ (1) and (2), ——— (4)

$$\boxed{E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{R}}$$

and
$$V = -\frac{\lambda}{2\pi\epsilon_0} \log_e R + \text{Constant} \qquad \text{——— (5)}$$

<u>Case (iii)</u> :– When the point P lies inside the cylinder i.e. $x < R$.

The cylinder is conducting, the total charge lies only on its outer surface. There is no charge within it. Hence the electric flux through the Gaussian surface will be zero.

$$\boxed{E = 0} \qquad \text{——— (6)}$$

Note that if the cylinder is hollow then also there will be no charge within

so that on its surface.